

Fracture toughness in fibrous materials

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In the present paper, a fiber bundle model in $(1+1)$ dimensions that simulates the rupture process of a fibrous material pulled by an uniaxial force F is analyzed. In this model the load of a broken fiber is shifted in equal portions onto the nearest unbroken fibers. The force-displacement diagram is obtained for several traction velocities v and temperatures t . Also, it is shown how the fracture toughness K_c changes with the traction velocity v and with the temperature t . In this paper it is shown that the rupture process is strongly dependent on temperature t and on velocity v .

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I. INTRODUCTION

Research needs in fracture mechanics are quite varied and still pose a formidable task for engineers and scientists. When a load sufficiently large is applied on a material, it fractures in a process that depends on several factors, such as, the external conditions (temperature, traction velocity, humidity etc.). The fracture of a material can be classified in two categories, brittle and ductile [1,2]. These two categories are not solely functions of the material properties but depend also on temperature and traction velocity. The brittle fracture generally occurs at low temperatures and/or high velocities, while the ductile fracture occurs at high temperatures and/or low velocities.

When a material is pulled by an uniaxial force F , it experiences a displacement δ . The force-displacement diagram provides important information about the fracture process and can be easily obtained experimentally. In this diagram one can detect a linear region in which the force F increases proportionally to the displacement δ , obeying Hooke's law. In this region, the mechanical response of the material is reversible, i.e., if the force is reset to zero the material returns to exactly the original shape. It is also observed that, if the force is increased beyond a certain critical value, the material enters the plastic region, where it does not return to the original shape when the force vanishes. If the rupture of the material occurs in the linear region it is called brittle and if the fracture occurs in the plastic region it is called ductile. Another important information obtained from the F versus δ diagram is the fracture toughness, i.e., the amount of energy needed to fracture the material. The fracture toughness can be evaluated from the area below the F versus δ curve. Experimental results [1,3,4] show that the brittle fracture consumes less energy than the ductile fracture.

The fracture properties of disordered materials is a subject of great interest because the presence of disorder is an important feature that determines the rupture process [5]. To analyze the rupture process of disordered materials several models were proposed, among which is the well-known fiber bundle model (FBM) [6–11], created from the pioneer work of Daniels [12]. In the FBM a set of fibers is distributed on a supporting lattice forming a fiber bundle. The fiber bundle is fixed at both extremes by two parallel plates, one of them is fixed and on the other an external load is applied. The FBM

can be time independent (static FBM) [12,13] or not (dynamic FBM) [8–11,14–16]. In the static model, to each fiber of the bundle is assigned a strength threshold from a probability distribution and if the applied load exceeds this threshold value the fiber breaks. In the dynamic model, each fiber is assumed to have a lifetime under a given load history, and it breaks because of fatigue. An important factor in the definition of the FBM is the load-sharing rules, which describe how the load of a broken fiber is transferred to the unbroken ones. In equal load-sharing (ELS) models the load carried by a broken fiber is equally distributed among the unbroken fibers of the bundle. In local load sharing (LLS) the load of a broken fiber is transferred only to its nearest neighbors.

In 1994, Bernardes and Moreira introduced an equal load-sharing FBM to simulate fractures in fibrous materials that is sensitive to external conditions, traction velocity and temperature [17]. In this work they obtained fracture energy (toughness) versus temperature diagrams for several traction velocities. Then, they concluded that the higher the traction velocity, the higher is the fracture toughness of the process. These results indicate that a brittle fracture consumes more energy than a ductile one, in marked disagreement with experimental results. In this paper, a FBM with local load sharing is studied in order to analyze the rupture process of a fibrous material pulled by a force F with a constant velocity v . The main goal is to obtain the force-displacement $F(\delta)$ diagram for several traction velocities v and evaluate the fracture toughness involved in the rupture process. It is investigated also how the fracture toughness K_c changes with the traction velocity v and with the temperature t .

II. MODEL

The present model was inspired in the one studied by Bernardes and Moreira [17]. It consists of a bundle of N_0 parallel fibers, all with the same elastic constant, k , distributed on a unidimensional lattice. The fiber bundle is fixed at both extremes by two parallel plates, one of them is fixed and the other pulled by an uniaxial force F with a constant velocity v . The force F on the fiber bundle is defined as

$$F = Nk\delta, \quad (1)$$

where δ is the displacement and N is the number of unbroken fibers. At each time step the bundle experience an increase $\Delta\delta = v \times \tau$ in the displacement, where in our units $\tau = 1$. In the model presented here the fiber failure probability depends on the applied load σ . The load ($\sigma = F/N$) is the external force F on the bundle divided by N , the total number of unbroken fibers in the bundle, therefore, $\sigma = k\delta$. Since the model is of LLS type, an unbroken fiber i supports a load σ_i given by

$$\sigma_i = \left(1 + \frac{j}{2}\right)\sigma, \quad (2)$$

where j is the number of broken fibers on both sides of the fiber i . The failure probability of a fiber i is given by a Weibull distribution usually used in materials science [5,18–20]

$$P_i(\sigma_i) = 1 - \exp\left[\frac{-(\sigma_i)^\rho v}{t}\right], \quad (3)$$

where t is temperature, ρ is the Weibull modulus, which controls the degree of disorder in the system, and v is the traction velocity. This definition of the failure probability is different from that used by Bernardes and Moreira [17] that computed the failure probability from the elastic energy of a fiber.

At the beginning of the simulations all fibers are complete and submitted to the same load σ ($j=0$). At each time step fibers are randomly chosen from a set of $N_q = qN_0$ unbroken fibers. The number q represents a percentage of fibers and allow us to work with any system size. Then, using Eqs. (2) and (3), the fiber failure probability P_i is evaluated and compared with a random number r in the interval $[0,1)$. If $r < P_i$ the fiber breaks and then the neighboring unbroken fibers are tested. This procedure describes the propagation of a crack through the fiber bundle in the direction perpendicular to the applied force. The process of propagation stops when the test of the probability does not allow rupture of any other fiber on the border of the crack or when the crack meets another already formed crack. The same cascade propagation is attempted by choosing another fiber of the set N_q . After all the N_q fibers have been tested, the bundle is pulled to a new displacement $\Delta\delta$ and all the rupture process is restarted. The simulation terminates when all the fibers of the bundle are broken, i.e., when the bundle is divided into two parts.

III. RESULTS

In order to verify the influence of the temperature t and velocity v in the rupture process of a material, the simulations were performed considering $N_0 = 1 \times 10^4$ fibers, elastic constant $k=1$, and Weibull modulus $\rho=2$. The simulations were averaged over 1000 statistically independent samples.

Initially, the force-displacement diagrams were obtained in order to verify the influence of temperature t and velocity v on the fracture process. Figure 1 shows the force-displacement diagram $F(\delta)$ for three velocities v and two different temperatures t . In Fig. 1(a) the results obtained for

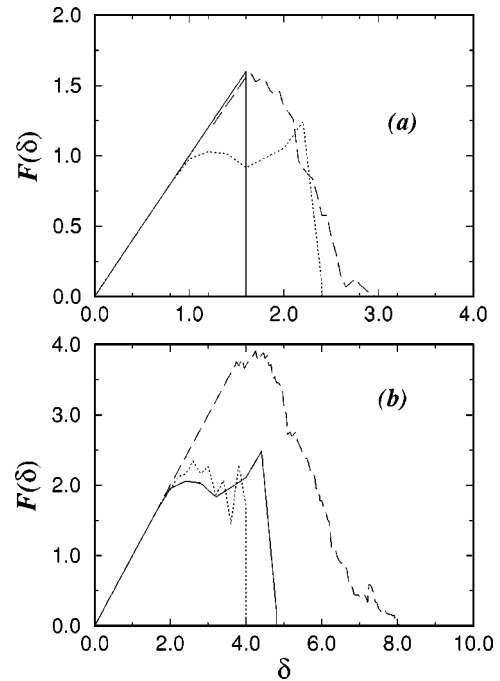


FIG. 1. Force F as a function of the displacement δ for three different velocities v and two temperatures t . In (a) we have $t = 0.5$ and in (b) $t = 4.0$ arbitrary units. $v = 0.4$ (solid line), $v = 0.2$ (dotted line), and $v = 0.05$ arbitrary units (long dashed).

$t = 0.5$ are shown. Note that for $v = 0.4$ the relation between the force F and the displacement δ is purely linear. This behavior is characteristic of a brittle fracture, where the rupture occurs due to the appearance of big cracks in the material. For low and intermediate velocities the relation between the force F and the displacement δ is not purely linear and in this case the fracture occurs in the brittle-ductile transition or in the ductile region. From Fig. 1(a) one can see that the lower the velocity, the greater the area below the force-displacement curve.

In Fig. 1(b) the results were obtained for a temperature $t = 4.0$. Now, for any velocity v , the force-displacement diagram is not purely linear, i.e., for this temperature and these sets of velocities the fracture will not be brittle. In order to better understand the influence of the temperature t and velocity v on fracture process, we will discuss how the fracture toughness K_c is influenced by the temperature t and the velocity v .

Figure 2 shows a log-log plot of the fracture toughness K_c as a function of the temperature t for three different velocities, $v = 0.002$, $v = 0.02$, and $v = 0.05$. Note that the fracture toughness K_c increases linearly with the increase of the temperature t , indicating a power law

$$K_c \approx t^\alpha, \quad (4)$$

where α is an exponent that depends on the velocity. So, the higher the temperature t , the more energy will be absorbed before a catastrophic rupture occurs. Also, Fig. 2(a) shows that the higher the velocity v , the lower the fracture toughness K_c , i.e., a smaller quantity of energy will be spent in the

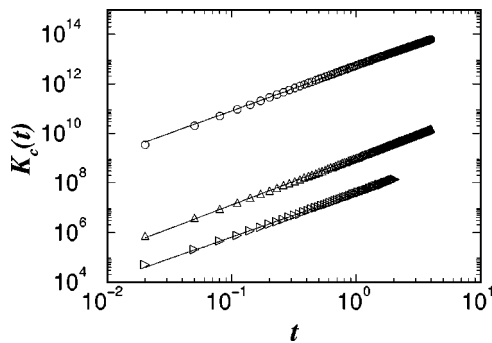


FIG. 2. Log-log plot of the fracture toughness K_c as a function of the temperature t for three different velocities, $v=0.002$ (circles), $v=0.02$ (up triangles), and $v=0.05$ (right triangles).

fracture process. In Fig. 3 this fact is shown more clearly. It shows a log-log diagram of the fracture toughness K_c versus the velocity v for three temperatures. Note that the fracture toughness decreases with the increase of the velocity.

The results obtained agree with experimental data obtained in fracture mechanics [1,3,4]. It is well known that the failure of materials has a strong dependence on temperature and velocity, for example, as is the case for the failure behavior of polymers. At very low temperatures they fracture due to being brittle and consume little energy during the rupture process [1,5,21]. When the temperature is increased above the critical temperature t_c , the polymer undergoes a transition to rubberlike behavior in which the material can be elastically stretched over several times its initial size. In this region the fracture process is slow and consumes very much energy. Also, the rupture behavior of polymers is strongly dependent on the speed in which the elongation takes place.

IV. CONCLUSION

In conclusion, we studied a model for fracture on fibrous materials in $(1+1)$ dimensions that simulates a rupture process sensitive to temperature t and to velocity v . It is well known that the fracture toughness, i.e., the energy (work) consumed to break the material is strongly dependent on temperature and on traction velocity. At low temperatures and/or high velocities the fracture toughness is lower than

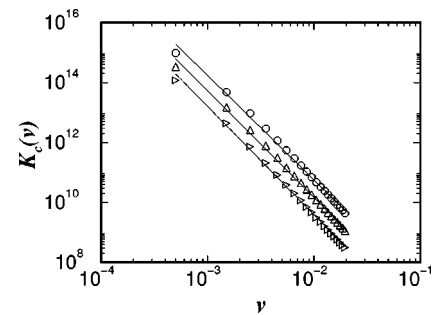


FIG. 3. Log-log plot of the fracture toughness K_c versus the velocity v for three different temperatures, $t=2.0$ (circles), $t=1.0$ (up triangles), and $t=0.5$ (right triangles).

that in high temperatures and/or low velocities.

In Ref. [17] Bernardes and Moreira, used a fiber bundle model for which the fracture toughness is sensitive to temperature t and to velocity v . However, their results do not agree with experimental observations. In the present paper, it was studied a similar model to the one used by Bernardes and Moreira [17] and the force-displacement diagrams for three different velocities and two temperatures were obtained. In these diagrams, one can observe two regions dependent on temperature t and velocity v , an elastic and a plastic region. In the elastic region the force F is linearly proportional to displacement δ . In the plastic region the force F is not linearly proportional to the displacement δ and with the increase in δ it reaches a maximum value, beyond which it decreases. The area below the force-displacement curve give us the toughness K_c and depends on the temperature and velocity. The results obtained in this work show that the fracture toughness K_c increases with the increase of the temperature t and decreases with the increase of the velocity v . These results are in agreement with the experimental observations.

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